HCU-003-001308

B. Sc. (Sem. III) (CBCS) Examination

October/November - 2017

Mathematics : BSMT-301(A)

(Linear Algebra, Calculus & Theory of Equations)

(Old Course)

Faculty Code : 003
Subject Code : 001308

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions carry equal marks.

(2) All questions of **SECTION-A** carry equal marks and each question of **SECTION-B** carry 25 marks.

(3) Write answer of each section in your main answer sheet.

SECTION - A

1 Answer the following short questions : **20**

(1) Define : Subspace of a vector space.

(2) Examine whether {(1,0,0), (0,1,0), (0,0,0)} is linearly independent or not.

(3) "If W is a subspace of a finite dimensional vector space V and $\dim W = \dim V$ then $W = V$" : True/False.

(4) If W_1 and W_2 are two subspaces of a finite dimensional vector space V then $\dim(W_1 + W_2) =$ ____________.

(5) Define : Kernel of a linear transformation.

(6) Let $L(U, V)$ be the set of all linear transformations from a vector space U to a vector space V, $\dim U = m$ and $\dim V = n$. What is $\dim(L(U, V))$?

(7) Define : Idempotent linear transformation.

(8) Define : Eigen value of a linear transformation.

(9) If a series $\sum u_n$ is convergent then $\lim_{n\to\infty} u_n =$ _______.

(10) For which values of r the series $\sum_{r=0}^{\infty} r^n$ is convergent ?

(11) Examine whether the series $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)$ is convergent or divergent.

(12) Examine whether the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ is convergent or not.

(13) Determine an interval in which a positive real root of the equation $x^3 - 2x - 5 = 0$ lies.

(14) Write formula to find first approximation to the root of the equation $f(x) = 0$ using false position method.

(15) Write iterative formula to find reciprocal of a positive number N using Newton-Raphson method.

(16) State the condition for convergence of method of successive approximations.

(17) Define : Node.

(18) State condition for a curve $y = f(x)$ to be convex downwards.

(19) Write the formula to find radius of curvature for the curve represented by $x = f(t),\ y = g(t)$, where t is a real parameter.

(20) Curvature of any straight line is zero : True/False.

SECTION - B

2 (a) Attempt any **three** : **6**

(1) Prove that intersection of two subspaces of a vector space V is also a subspace of V.

(2) Check whether the set $A = \{(1, 1, -1), (1, 0, 1), (1, 1, 0)\}$ is linearly independent or not.

(3) Let U and V be two vector spaces over R. If θ and θ' are zero vectors of U and V respectively then prove that :

(i) $T(\theta) = \theta'$

(ii) $T(-u) = -T(u), \forall u \in U$.

(4) Find nullity of the linear transformation $T : R^3 \to R^2$ defined as $T(x, y, z) = (x, x + y, y)$.

(5) Test for convergence the series $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\dots$

(6) State Rabbe's Test for convergence of a series.

(b) Attempt any three : **9**

(1) Find coordinates of $3+7x+2x^2 \in P_2(R)$ with respect to the basis $\{1-x, 1+x, 1-x^2\}$.

(2) Extend $A=\{(1,1,0),(2,0,0)\} \subset R^3$ to form a basis of R^3.

(3) Prove that $T: R^3 \to R^3$ defined by

$T(x, y, z)=(x-y, y-z, z-x), \quad (x, y, z) \in R^3$ is a linear transformation.

(4) If $T: R^3 \to R^2, T(a, b)=(a,-b),(a, b, c) \in R^3$ is linear transformation then find $[T; B_1, B_2]$, where $B_1=\{(1,1),(1,0)\}$ and $B_2=\{(2,3),(4,5)\}$ are basis of R^2.

(5) Examine convergence of the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}+\sqrt{n+1}}\right).$$

(6) Determine the interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{x^n}{n}.$$

(c) Attempt any two : **10**

(1) Show that the set V = (a, b) | a, b $\in$ R, b > 0} is a vector space over R under the operations

(a, b) + (c, d) = (a + c, bd), and α(a, b) = (αa, b^{α}).

(2) State and prove Rank-Nullity theorem.

(3) Find eigenvalues of the linear transformation

T : $R^2 \to R^2$ defined as

T(x, y) = (3y, 2x – y) by considering the basis

$B = \{(1, 0), (0, 1)\}$

(4) Test for convergence of the series

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}+\frac{7}{4\cdot 5\cdot 6}+\ldots$$

(5) Test for convergence of the series

$$1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\ldots+\frac{x^n}{n^2+1}+\ldots$$

Also find interval of-convergence (if exist).

3 (a) Attempt any three : **6**

(1) For f(x) = $x^3 - 6x^2 + 11x - 7$, find f'(1).

(2) Find an equation whose roots are with opposite signs of the roots of the equation

$x^3 - 6x^2 + 11x - 6 = 0$.

(3) Check whether the function $f(x) = e^x$ is concave upwards or concave downwards.

(4) If the curve $y = x^3 + ax^2 + bx$ has a point of inflexion at point (3,–9) then find the values of a and b.

(5) Find asymptotes parallel to coordinate axes for the curve $(x^2 + y^2)x - ay^2 = 0$.

(6) Prove that origin is cusp for the curve $y^2 = 2x^2y + x^3y + x^3$.

(b) Attempt any three : **9**

(1) Explain graphical method to find an approximate root of equation $f(x) = 0$.

(2) Using Newton-Raphson method, find the value of $\sqrt{101}$ correct to four decimal places.

(3) Obtain the transformed equation by diminishing the roots of the equation $x^4 + 3x^3 - 2x^2 - 4x - 3 = 0$ by 3.

(4) Prove that the curvature at any point on a circle is constant.

(5) Find all the asymptotes of the curve $y^3 - x^2(6 - x) = 0$.

(6) Find radius of curvature at origin of the curve $x^3 - 2x^2y + 3xy^2 - 4y^3 + 5x^2 - 6xy + 7y^2 - 8y = 0$.

(c) Attempt any two : **10**

(1) Find an approximate real root of the equation $x^3 - x - 11 = 0$ correct to three decimal places using bisection method.

(2) Find an approximate real root of the equation

$x\log_{10} x - 1.2 = 0$ correct to three decimal places using false position method.

(3) Find an approximate real root of the equation

$x^3 - 5x + 3 = 0$ correct to four decimal places using Newton-Raphson method.

(4) Find radius of curvature at any point on the curve

$y^2 = 4ax$.

(5) Find multiple points on the curve

$x^3 + y^3 - 3x^2 - 3xy + 3x + 3y - 1 = 0$ and

determine its type.